Selection of papers of Prof. Aldo de Luca and top achievements

We list some publications which had a noteworthy resonance at international and national level:

1. Decision equation for binary systems. Application to neuronal behaviour, Kybernetik **3** (1966) 33-40 (with E. R. Caianiello) [n. 1 of the list]

In this paper we study the behavior of a single neuron subjected to an external stimulus, described by an integral equation of Volterra type with a temporal shift.

The paper has been cited by W. McCulloch (the father of Automata theory) in his article: *Logic and closed loops for a computer junked to Mars* published in the volume "Neural Networks" (Springer-Verlag Berlin 1967 pp. 65–91) and was of interest to neurophysiologist, nobel prize, R. Granit.

2. Reverberations and Control of neural networks, Kybernetik **4** (1967) 10-18 (with E. R. Caianiello and L. M. Ricciardi) [n. 3 of the list]

In this paper (see also the papers 5, 6, 9, 11, 12) we study the dynamical behaviour of an autonomous network of linear threshold elements. In particular, we are interested in the study of reverberations or cycles of the net. We give some conditions on the matrix of coupling coefficients among neurons under which there exist some constants of the motion and the length of cycles can be suitably limited.

The paper had a great resonance at international level. The main results are reported in various articles and volumes on neural networks (see, for instance, the volume of A. C. Scott, "Neurophysics", 1977, John Wiley).

3. A definition of a non-probabilistic entropy in the setting of fuzzy sets theory, Information and Control **20** (1972) 301-31 (with S. Termini) [n.14 of the list]

We introduce in an axiomatic way a non-probabilistic entropy in the setting of "Fuzzy sets" theory of L. Zadeh. This entropy can be interpreted as a measure of the total amount of uncertainty that one has in making a decision (1 or 0) in a universe of objects described by a fuzzy set.

This paper was defined by various authors a pioneristic paper. In fact, it was the basic paper of numerous other contributions on the same subject.

The paper was reprinted in various volumes (for instance, Dubois, Prade, and Yager ed.s "Readings in Fuzzy sets for Intelligent Systems, Morgan Kaufmann Publ., Inc.). Moreover, the Department of Army of United States was very interested in this kind of entropy.

On this subject I have published with S. Termini and R. Capocelli the papers 13, 15–17, 21 of the publication list. I point out the item 'Entropy measures in the theory of fuzzy sets' of the Encyclopedia of Systems and Control of Pergamon Press written with S. Termini. In 1980, I collected more than 300 quotations of the previous paper. At the present google scholarship reports 1770 citations.

4. Minimal complete sets of words, Theoretical Computer Science **12** (1980) 325-332 (with J. M. Boë and A. Restivo) [n.27 of the list]

This paper is one of a series of articles on variable length codes (see 20, 22-26, 29-31, 71, 81,82, 84, 89 of the enclosed list). We prove a property of fundamental importance for complete sets and maximal codes (a code is maximal if it is not properly included in another code on the same alphabet, a set is complete if any word is a factor of some word of X^*). The main result is that if a set X of words is non-dense (for instance, finite) and complete and there exists a positive Bernoulli distribution μ such that $\mu(X) = 1$, then X is a maximal code. At the present this is a classic result in Theory of codes. It is reported in the Universitary handbook of Algebra of P. Cohn (2nd edition) vol. 2, 1989, John Wiley (see, Theorem 4.8, pp. 403-404).

5. A characterization of strictly locally testable languages and its application to subsemigroups of a free semigroup, Information and Control **44** (1980) 300-319 (with A. Restivo) [n. 26 of the list]

In this paper we give a characterization, both algebraic and combinatorial, of the family of languages 'locally testable' in the sense of McNaughton and Papert.

The main results have been quoted by various authors (for instance, J. E. Pin, "Varieties of Formal Languages", North Oxford Academic, London and Plenum, New York 1986) and, moreover, have been utilized by T. Head to describe the language of the 'splicing sequences' of DNA (T. Head, Formal language theory and DNA: an analysis of the generative capacity of specific recombinants behaviors, Bull. Math. Biology 49 (1987) 737-759). The paper is also quoted in the Bibliography of the Chapter: T. Head, G. Paun and D. Pixton: Language Theory and Molecular Genetics: Generative Mechanisms suggested by DNA recombination, published in vol. 2 of the Handbook of formal languages (Rozenberg and Salomaa ed.s), Springer 1997.

6. On non-counting regular classes, Theoretical Computer Science **100** (1992) 67-104. (with S. Varricchio) [n. 52 of the list]

In this paper (see also the papers 83 and 99 of the enclosed list) we show the validity for $n > 4$ of the conjecture by J. Brzozowski, formulated in 1969, relative to the regularity of non-counting classes of order n. In other terms the set of words of any non-counting class of order n over any finite alphabet is a language recognizable by a finite automaton. This problem appeared in a list of 10 problems considered of fundamental importance for the theory of Automata and Formal languages. The resolution of the conjecture for $n > 4$ required more than two years of intense research work and the proof is about 50 pages long. This result was considered 'magnificent' by the reviewer of Mathematical review. An important consequence of it was the proof of decidability for $n > 4$ of the 'word problem' in the Burnside variety defined by the equation $x^n = x^{n+1}$. Subsequently, the Brzozowski conjecture was proved for $n = 4$ by A. Pereira do Lago and for $n = 3$ by V. Guba. It is still open the case $n = 2$ (the case $n = 1$ is trivial).

7. On a conjecture of Brown, Semigroup Forum **46** (1993)116-119 (con S. Varricchio) [n. 54 of the list]

In this short note we prove a conjecture posed by T. C. Brown in the paper (T. Brown, Cancellation in semigroups in which $x^2 = x^3$, Semigroup Forum 41 (1990) 49-53). The proof uses a very special technique of semigroup theory

which was very much appreciated by Brown.

8. Sturmian words: Structure, Combinatorics, and their arithmetics, Theoretical Computer Science **183** (1997) 45-82 [n.62 of the list]

In this paper we show new original results concerning the structure, the combinatorics, and the arithmetics of the set PER of finite words having two periods p and q which are coprime and such the length $|w|$ of w satisfies the condition $|w| = p + q - 2$. The set PER is the kernel of the theory of Sturmian words (an infinite word is Sturmian if has the minimal value for the subword complexity without being ultimately periodic). Indeed, PER equals the set of the palindromic prefixes of all standard Sturmian words; moreover, the set of all finite factors of all standard Sturmian words coincides with the set of factors of PER (see, also the papers 28, 58, 59, 60, 61, 80, 84, 86, 87, 88, 89, 90, 93, 103, 105, 108 of the enclosed list). In this paper we prove that it is possible to construct all standard Sturmian word by iterating the application of the operator of palindrome closure, defined for any word w as the shortest word having w as a prefix. Moreover it is show the strong relation existing between Sturmian words and the theory of continued fractions.

The main results of this paper, very much quoted in the literature, have been used by several authors (see Berstel and Séébold, Chapter 2, Sturmian words in "Algebraic Combinatorics on words", Cambridge University Press,2002). Moreover, some authors (X. Droubay, J. Justin and G. Pirillo, Episturmian words and some constructions of de Luca and Rauzy, Theoretical Computer Science, 255 (2001)539– 553; J. Justin and G. Pirillo, Episturmian words and episturmian morphisms) have extended the operation of palindrome closure to the case of alphabet containing more than two letters introducing a class of infinite words, called *standard episturmian*, larger than the class of standard Sturmian words. Finally, this operation of iterated palindrome closure can be extended to the case of general involutory antimorphisms of a free monoid giving rise to larger classes of infinite words extending Sturmian and episturmian words (see papers 91, 92, 102 of the enclosed list).

9. Words and special factors, Theoretical Computer Science 259 (2001) 145–182 (with A. Carpi)[n.67 of the list]

Following a new approach to combinatorics on words introduced in 113 , we consider sets of factors of a given finite word which permit to reconstruct the entire word. The analysis is based on the notion of special factor. A factor u of a finite word w is called right (left) special if there exist two different letters x and y such that ux, uy (resp., xu, yu) are factors of w. A factor is bispecial if it is right and left special. A proper box of w is any factor of w of the kind asb with a, b letters and s a bispecial factor. The initial (resp., terminal box) of w is the shortest prefix (resp., suffix) of w which is an unrepeated factor. A box is called maximal if it not a proper factor of another box. The main result of the paper is the following theorem (maximal box theorem): *Any finite word is uniquely determined by the initial box, the terminal box and the set of maximal boxes.*

A consequence of this result is that a finite word w is uniquely determined by the knowledge up its factors up to the length $n = \max\{R_w, K_w\} + 1$, where K_w is the length of the terminal box and R_w is the minimal natural number for which in w there is no right special factor of length R_w.

These results have several applications both from theoretical and applicative point of view. Some extensions have been given to the case of languages and some non-linear structures such two-dimensional arrays and rational trees (see papers 70, 79). Moreover, in 72 we have introduced an efficient algorithm for the problem of *sequence assembly* which is a problem of fundamental importance in molecular Biology. Finally, some generalizations of periodic words have been proposed in 66. 68, 69, 76.

3 Top achievements

In my opinion at the top of the results obtained in my research activity is the proof with S. Varricchio of the regularity of non-counting classes for $n > 4$ (n. 6 of the selected papers). The main reason is the great difficulty of the problem (the solution required two years of intense work) and the fact that this problem was one of the most important open problems of Automata theory. The proof, which has not been simplified with the exception of some minor details, requires a lot of knowledge of theory of semigroups, automata theory, and combinatorics on words.

The second top achievement concerns Sturmian sequences (number 8 of the selected papers). It is at the base of several important developments and extensions of the theory. It has been quoted by several authors. (C. Reutenauer and others say that this is the 'Bible' on the subject of Sturmian sequences).

A third top result is the paper: Words and special factors (with A. Carpi) (n. 9 of the selected papers). The importance of this paper is the use of a new approach at combinatorics on words associating to each word some characteristic parameters related to repetitive structure of the word. This approach was introduced by myself in a previous paper [64], but in this paper with Carpi it is proved a theorem of great importance for the theory and the applications.

A fourth top result was the introduction (with S. Termini) of a non-probabilistic entropy in the setting of fuzzy sets theory. In fact, this was a pioneristic paper cited since 1972 by a great number of authors. In fact, it was the basic paper of numerous other contributions on the same subject and reprinted in various volumes.